Stochastic Flutter of a Panel Subjected to Random In-Plane Forces Part I: Two Mode Interaction

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The effect of random variation of in-plane load acting on panels in supersonic flow is examined. The analysis includes stochastic stability and response moments of linear and nonlinear panels. The response moment equations are genearted by using the Fokker-Planck equation approach. The mean-square stability boundaries and response moments are determined as functions of the spectral density of in-plane loads, aerodynamic pressure, air-to-structure mass ratio, and structural damping ratios. The nonlinear response is estimated by using a cumulant-neglect scheme. For equal modal damping coefficients, it is found that the damping stabilizes the panel in the sense of mean square. However, a paradoxical effect of the damping is found only for unequal damping coefficients where the effect is nonbeneficial. This observation is identical to the well-known results pertaining to the deterministic theory of panel flutter. The nonlinear response statistics are obtained in the time domain, the steady-state reveals that the response process is strictly stationary. This feature is believed to be first known for coupled systems with stiffness nonlinearity and is contrary the nonstationary response characteristics of dynamic systems with inertia nonlinearity.

Nomenclature

a = plate length
c = structural damping coefficient
D = $Eh^3/12(1-\nu^2)$ is the plate bending stiffness
E = modulus of elasticity
h = plate thickness
$\bar{m}$ = mass per unit area of the panel
M = Mach number = U_∞/a_s
$N_x(t)$ = external random in-plane load per unit span-wise length = $N_x^0 + n_x(t)$
U_∞ = gas-flow speed
x = stream-wise spatial coordinate
Δp_s = static component of gas pressure
$\Delta p(t)$ = random component of gas pressure
ν = Poisson's ratio
ρ_∞ = gas density

I. Introduction

PANEL flutter is a form of dynamic instability resulting from the interaction of the motion of the panel with the aerodynamic pressure. This interaction causes the differential equations of motion of the panel to be nonself adjoint. The aerodynamic pressure fluctuates randomly, and the plate responds as a linear filter. As the flow pressure increases above a critical flutter value, the plate motion changes from random into highly ordered with increasing amplitude. However, when the panel is subjected to a compressive, in-plane load, in addition to the random aerodynamic pressure, the response may undergo complex motions,[1] known as chaos.[2]

The effect of in-plane load has been treated in the literature[3-5] as a static component for two- and three-dimensional panel flutter. Generally, tensile in-plane loads tend to stabilize panel response; whereas compressive loads tend to increase panel response. The compressive loads which are below the critical buckling load of the panel have only a small effect on panel response.[6] In most cases, the in-plane forces are varying with time due to local vibration of the surface panels. For harmonic, in-plane loading, Dzygadlo and Kaliski[7] and Dzygadlo[8] determined the stability-instability boundaries due to harmonic in-plane loading for subsonic and supersonic gas flows. When the panel is exposed to subsonic flow, it is found that the region of parametric instability becomes wider as the Mach number increases up to a critical value above which the instability region shrinks and leaves the frequency axis. Above this critical Mach number, the instability region is dominated mainly by self-excited vibration (flutter). The regions of parametric instability do not exist if the flow becomes supersonic only for the case of a panel with infinite length. For a panel with finite length exposed to supersonic flow, Dzygadlo found that the presence of air flow would change the location and shape of the regions of parametric instability. The air flow was found to create aerodynamic damping and coupling between panel normal modes. The Mach number causes a shrink in the first region of parametric instability. This region also shifts towards high values of excitation frequency and amplitude. This trend, however, is reversed for the second instability region. For lower values of Mach number, the second region is displaced away from the frequncy axis due to the effect of aerodynamic damping. As the Mach number increases, the instability region starts to expand and moves towards smaller values of excitation frequency and amplitude until the Mach number reaches a critical value M_{cr}, where the region becomes maximum and touches the frequency axis. For $M > M_{cr}$, the instability region shrinks again.

According to Fung,[9] the difference between the linear theory of panel flutter and experimental results is attributed to the fact that the experiment observes a stochastic forced oscil-

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lation due to turbulence under which several modes are excited. Another important reason is due to the nonlinear effects of both structural and aerodynamic forces. Very few attempts[4,6,10,11] have been made to examine panel flutter and its response to turbulent pressure fluctuations within the boundary layer. Although Eastep and McIntosh[4] considered nonlinear structural and aerodynamic nonlinearities, they linearized the governing equations when they considered aerodynamic loading. One of the most important observations in their deterministic analysis is that the two types of nonlinearities have different mechanisms. The nonlinear interaction between inplane stresses and transverse deformation provides a stabilizing influence on the panel motion in that it acts to restrain further deformation. The nonlinear aerodynamic loading, on the other hand, has a destabilizing effect in that it acts to increase any deformation. Vaicaitis et al.[6] and Olson[11] utilized a Monte Carlo simulation for the response analysis of a panel undergoing large deformations under turbulent boundary-layer pressure. They decomposed the pressure field acting on the panel into three components: the external flow (radiation) pressure, the internal (cavity) pressure, and the random pressure resulting from the boundary-layer pressure fluctuations. The random pressure is represented by a stationary multidimensional Gaussian process, which has a specified cross spectral density. This random process is then simulated by a series of cosine functions with weighted amplitudes, almost evenly spaced random frequencies, and random-phase angles, which are uniformly distributed between 0 and 2π. The effect of cavity on panel response was examined for subsonic and supersonic flow regimes. The cavity acts as an air-spring resisting the modes that tend to compress the air, thus raising their modal frequencies. For subsonic flow, the response is reduced substantially by the presence of the cavity. However, for supersonic flow, flutter occurred at a lower dynamic pressure with a cavity than for the case without a cavity. This is due to the fact that the first mode frequency was raised by the aerodynamic spring effect producing an earlier coupling between the first- and second-mode frequencies and thereby induced flutter. The increase of the boundary-layer intensity is found to modify significantly the plate response especially for low-air pressure parameter. Other numerical techniques such as the finite element approach have been employed for computing nonlinear flutter characteristics of panels in supersonic flow.[12,13]

This three-part paper examines the stochastic flutter of a panel exposed to supersonic flow in two dimensions. The first part deals with a two-mode interaction utilizing Gaussian closure scheme. Part II will examine the nonlinear response by using a non-Gaussian closure approach. The third part will treat three-mode interaction by using Gaussian and non-Gaussian closures. The response moment equations are generated by using the Fokker-Planck equation approach. The stability-instability boundaries are plotted as functions of the intensity of in-plane load, aerodynamic pressure, air-to-structure mass ratio, and structural damping ratios. The linear and nonlinear mean-square responses of the panel are also determined. The nonlinear response is estimated by using a cumulant-neglect scheme.

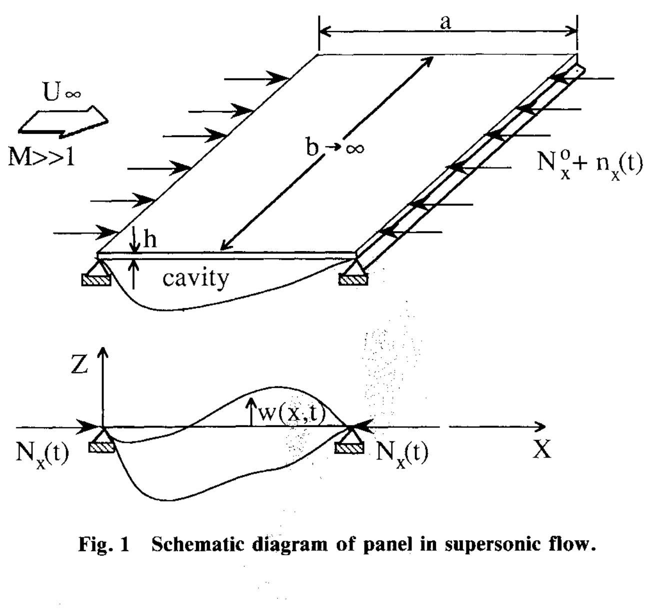

Fig. 1 Schematic diagram of panel in supersonic flow.

II. Analysis

Based on the quasisteady-state, supersonic theory, the deflection $w(x,t)$ of a two-dimensional panel (shown in Fig. 1) undergoing cylindrical bending is governed by the equation of motion[1-3,7,8]

$$\bar{m}\partial^2 w/\partial t^2 + D(1+c\partial/\partial t)\partial^4 w/\partial x^4 - \left\{N_x(t) + (Eh/2a)\int_0^a (\partial w/\partial x)^2\, dx\right\}\partial^2 w/\partial x^2 + \frac{\rho_\infty U_\infty^2}{M}\left\{\partial w/\partial x + \frac{1}{U_\infty}\partial w/\partial t\right\} = \Delta p_s + \Delta p(t) \qquad (1)$$

The random component of gas pressure $\Delta p(t)$ is characterized by a power spectral density for which empirical expressions[6,14-16] can be used or an assumed Markov field may be introduced. The use of empirical expressions requires numerical simulation of the equations of motion, and the Markov field approximation yields a set of differential equations for the response moments. In the present analysis, the Markov field approximation theory will be used. In solving Eq. (1), the panel deflection $w(x,t)$ is expanded in terms of normal modes. For the case of simply supported plate, $w(x,t)$ can be written in the form

$$w(x,t) = \sum_{n=1}^{\infty} a_n(t)\sin(n\pi x/a) \qquad (2)$$

The shape function in Eq. (2) satisfies the boundary conditions

$$w(0,t) = w(a,t) = \partial^2 w(x,t)/\partial x^2\,|_{x=0,a} = 0 \qquad (3)$$

Substituting Eq. (2) into Eq. (1) and applying Galerkin's method results in a general ordinary differential equation for mode n. This equation is written in terms of the following nondimensional parameters.

$$A_n = a_n/h,\ \tau = t\sqrt{(D/\bar{m}a^4)},\ \lambda = \rho_\infty U_\infty^2 a^3/MD$$
$$\mu = \rho_\infty a/\bar{m},\ R_x^0 = N_x^0 a^2/D,\ \Delta R_x(\tau) = n_x(\tau)a^2/D$$
$$R_x(\tau) = R_x^0 + \Delta R_x(\tau),\ \zeta_n = (c/a^2)(n\pi)^4\sqrt{(D/\bar{m})} \qquad (4)$$

The resulting equation is

$$A_n'' + \left[\zeta_n + \sqrt{\mu\lambda/M)}\right]A_n' + (n\pi)^2\left[(n\pi)^2 + R_x(\tau)\right]A_n + 2\lambda\sum_m \frac{nm}{n^2-m^2}\left[1-(-1)^{n+m}\right]A_m + 3(1-\nu^2)\left[\sum_r A_r^2(r\pi)^2\right](n\pi)^2 A_n = 2P_s[1-(-1)^n]/(n\pi) + 2P_{nr}(\tau) \qquad (5)$$

where a prime denotes differentiation with respect to the nondimensional time parameter τ. The P_s is the static pressure $=\Delta p_s a^4/Dh$, and $P_{nr}(\tau)$ is the random pressure of mode n.

For a two-mode interaction, the differential equations of the first two modes are

$$A_1'' + (\zeta_1 + \zeta\sqrt{\lambda})A_1' + [C_4 + W_x(\tau)]A_1 - C_2A_2 + C_1A_1(A_1^2 + 4A_2^2) = C_3P_s + 2P_{1r}(\tau) \tag{6a}$$

$$A_2'' + (\zeta_2 + \zeta\sqrt{\lambda})A_2' + [C_5 + 4W_x(\tau)]A_2 + C_2A_1 + 4C_1A_2(A_1^2 + 4A_2^2) = 2P_{2r}(\tau) \tag{6b}$$

where

$$C_1 = 3(1-\nu^2)\pi^4, \qquad C_2 = 8\lambda/3, \qquad C_3 = 4/\pi$$
$$W_x(\tau) = \pi^2\Delta R_x(\tau), \qquad C_4 = \pi^4(1 + R_x^0/\pi^2)$$
$$C_5 = 4\pi^4(4 + R_x^0/\pi^2), \qquad \zeta = \sqrt{(\mu/M)}$$

III. Differential Equations of Response Statistics

Equations (6) are nonlinear stochastic differential equations with parametric random coefficients. The random coefficients $W_x(\tau)$ and the external random pressure $P_{nr}(\tau)$ are assumed uncorrelated, Gaussian, wide-band random processes with zero means. The correlation functions of these processes are

$$R_x(\Delta\tau) = E\left[W_x(\tau)W_x(\tau+\Delta\tau)\right] = 2D_x\delta(\Delta\tau) \tag{7a}$$

$$R_{nr}(\Delta\tau) = E\left[P_{nr}(\tau)P_{nr}(\tau+\Delta\tau)\right] = 2D_{nr}\delta(\Delta\tau), \quad n = 1,2 \tag{7b}$$

where $2D_x$ and $2D_{nr}$ are the spectral densities of the white noise processes $W_x(\tau)$ and $P_{nr}(\tau)$, respectively, and $\delta(\Delta\tau)$ is the Dirac-delta function. The white noise processes $W_x(\tau)$ and $P_{nr}(\tau)$ can be expressed as the formal derivative of the Brownian motion, i.e.,

$$W_x(\tau) = \frac{\sigma_x\, dB_x(\tau)}{d\tau}, \quad \text{and} \quad P_{nr}(\tau) = \frac{\sigma_{nr}\, dB_{nr}(\tau)}{d\tau}$$

where $B_i(\tau)$ are Brownian motion processes, $i = x$, nr. Through the coordinate transformation $\{A_1, A_1', A_2, A_2'\} = \{X_1, X_2, X_3, X_4\}$, Eqs. (6) may then be written in terms of Ito stochastic integrals in the state vector form

$$dX_1 = X_2\, d\tau \tag{8a}$$

$$dX_2 = \left\{-\zeta_1X_2 - \zeta\sqrt{\lambda}X_2 - C_4X_1 + C_2X_3 - C_1X_1(X_1^2 + 4X_3^2)\right\} d\tau - dB_x(\tau)X_1 + C_3P_s\, d\tau + 2dB_{1r}(\tau) \tag{8b}$$

$$dX_3 = X_4\, d\tau \tag{8c}$$

$$dX_4 = -\left\{\zeta_2X_4 + \zeta\sqrt{\lambda}X_4 + C_5X_3 + C_2X_1 + 4C_1X_3(X_1^2 + 4X_3^2)\right\} d\tau - 4dB_x(\tau)X_3 + 2dB_{2r}(\tau) \tag{8d}$$

The Fokker-Planck equation for the Ito Eqs. (8) is

$$\frac{\partial}{\partial\tau}p(X,\tau) = -\frac{\partial}{\partial X_1}X_2p(X,\tau) - \frac{\partial}{\partial X_3}X_4p(X,\tau) + \frac{\partial}{\partial X_2}\left\{\zeta_1X_2 + \zeta\sqrt{\lambda}X_2 + C_4X_1 - C_2X_3 + C_1X_1(X_1^2 + 4X_3^2) - C_3P_s\right\}p(X,\tau) + \frac{\partial}{\partial X_4}\left\{\zeta_2X_4 + \zeta\sqrt{\lambda}X_4 + C_5X_3 + C_2X_1 + 4C_1X_3(X_1^2 + 4X_3^2)\right\}p(X,\tau) + \frac{\partial^2}{\partial X_2^2}(D_xX_1^2 + 4D_{1r})p(X,\tau) + 8D_x\frac{\partial^2}{\partial X_2\partial X_4}X_1X_3p(X,\tau) + 4\frac{\partial^2}{\partial X_4^2}(4D_xX_3^2 + D_{2r})p(X,\tau) \tag{9}$$

It is difficult to solve this equation for the response transition probability density $p(X,\tau)$ even for the stationary case. Instead, one can generate a set of first-order differential equations describing the evolution of response statistical moments.[17] This set is obtained by multiplying both sides of Eq. (9) by the scalar function $\Phi(X_1^{k1}X_2^{k2}X_3^{k3}X_4^{k4})$ and integrating by parts over the entire space $-\infty < X < \infty$. This results in the general moment equation

$$\begin{aligned} m'_{k1k2k3k4} &= K_1m_{k1-1,k2+1,k3,k4} - (\zeta_1K_2 + \zeta_2K_4)m_{k1k2k3k4} \\ &- \zeta\sqrt{\lambda}(K_2 + K_4)m_{k1,k2,k3,k4} + C_3P_sK_2m_{k1,k2-1,k3,k4} \\ &- C_4K_2m_{k1+1,k2-1,k3,k4} + C_2K_2m_{k1,k2-1,k3+1,k4} \\ &- C_1K_2m_{k1+3,k2-1,k3,k4} - 4C_1K_2m_{k1+1,k2-1,k3+2,k4} \\ &+ K_3m_{k1,k2,k3-1,k4+1} - C_5K_4m_{k1,k2,k3+1,k4-1} \\ &- C_2K_4m_{k1+1,k2,k3,k4-1} - 4C_1K_4m_{k1+2,k2,k3+1,k4-1} \\ &- 16C_1K_4m_{k1,k2,k3+3,k4-1} + D_xK_2(K_2-1)m_{k1+2,k2-2,k3,k4} \\ &+ 4D_{1r}K_2(K_2-1)m_{k1,k2-2,k3,k4} \\ &+ 4D_{2r}K_4(K_4-1)m_{k1,k2,k3,k4-2} \\ &+ 8D_xK_2K_4m_{k1+1,k2-1,k3+1,k4-1} \\ &+ 16D_xK_4(k_4+1)m_{k1,k2,k3+2,k4-2} \end{aligned} \tag{10}$$

where the moment

$$m_{k1,k2,k3,k4} = \iiiint_{\infty}^{-\infty} X_1^{k1}X_2^{k2}X_3^{k3}X_4^{k4}p(X,\tau)\, dX_1\, dX_2\, dX_3\, dX_4 = E[X_1^{k1}X_2^{k2}X_3^{k3}X_4^{k4}]$$

and $E[\ldots]$ denotes expectation.

It is possible to generate a set of moment equations of any order N where $N = K_1 + K_2 + K_3 + K_4$. It is obvious that for the linear analysis (by setting $C_1 = 0$), the moment equations are consistent, i.e., they do not include moment terms of higher orders. In this case, one can solve for the response and stability of response moments. On the other hand, if the nonlinearity is retained, the moment equations become infinitely coupled and must be closed by an appropriate closure scheme for stability and response analyses.

IV. Random Response and Stability

A. Linear Two-Mode Coupling

First-Order Moments

The differential equations of first-order response moments are

$$\begin{aligned} m'_{1000} &= m_{0100} \\ m'_{0100} &= -(\zeta_1 + \zeta\sqrt{\lambda})m_{0100} + C_3P_s - C_4m_{1000} + C_2m_{0010} \\ m'_{0010} &= m_{0001} \\ m'_{0001} &= -(\zeta_2 + \zeta\sqrt{\lambda})m_{0001} - C_2m_{1000} - C_5m_{0010} \end{aligned} \tag{11}$$

The steady-state response of the first moments is obtained by setting the left sides of Eq. (11) to zero. The solution is

$$\begin{aligned} m_{0100} &= m_{0001} = 0 \\ m_{1000} &= C_3C_5P_s/[C_4C_5 + C_2^2] \\ m_{0010} &= -C_2C_3P_s/[C_4C_5 + C_2^2] \end{aligned} \tag{12}$$

This solution corresponds to the static buckling case, which shows that the two modes have zero-mean velocity response; whereas the mean displacements are nonzero and depend on the static and dynamic gas pressures and the in-plane, static load component R_x^0. The mean displacements of the two modes have opposite signs and are independent of the random parametric load.

The stability of the steady-state solution can be examined by introducing a perturbation to Eq. (12), i.e.,

$$\{m_{k1,k2,k3,k4}\} = \{m^0_{k1,k2,k3,k4}\} + \{\delta_{k1,k2,k3,k4}\}\exp(\Omega\tau) \quad (13)$$

where the vector $\{m^0_{k1,k2,k3,k4}\}$ represents the steady-state solution of Eq. (12), and the vector $\{\delta_{k1,k2,k3,k4}\}$ represents a set of small perturbations. Substituting Eq. (13) into Eq. (11) gives

$$\Omega\{\delta_{k1,k2,k3,k4}\} = [A]\{\delta_{k1,k2,k3,k4}\} \quad (14)$$

The stability of the first-order moments is determined from the nature of the eigenvalues of the matrix $[A]$. If any real part of Ω is positive, the panel is said to be unstable in the mean. The eigenvalues of $[A]$ are determined from the fourth-order polynomial

$$\Omega^4 + a_3\Omega^3 + a_2\Omega^2 + a_1\Omega + a_0 = 0 \quad (15)$$

where

$$a_3 = \zeta_1 + \zeta_2 + 2\zeta\sqrt{\lambda}$$

$$a_2 = C_5 + C_4 + (\zeta_1 + \zeta\sqrt{\lambda})(\zeta_2 + \zeta\sqrt{\lambda})$$

$$a_1 = C_5(\zeta_1 + \zeta\sqrt{\lambda}) + C_4(\zeta_2 + \zeta\sqrt{\lambda})$$

$$a_0 = C_4C_5 + C_2^2$$

The Hurwitz stability criterion is applied to the characteristic Eq. (15), and it is found that the coefficient a_1 gives the instability condition which corresponds to the Euler buckling load of the first mode. This means that if R_x^0/π^2 is less than -1 (in-plane compression), the panel is unstable in the mean regardless of the aerodynamic pressure parameter λ.

Second-Order Moments

The differential equations of the response second moments are

$$m'_{2000} = 2m_{1100} \quad (16a)$$

$$m'_{0200} = -2(\zeta_1 + \zeta\sqrt{\lambda})m_{0200} + 2D_x m_{2000} + 8D_{1r} + 2C_3P_s m^0_{0100} - 2C_4 m_{1100} + 2C_2 m_{0110} \quad (16b)$$

$$m'_{0020} = 2m_{0011} \quad (16c)$$

$$m'_{0002} = -2(\zeta_2 + \zeta\sqrt{\lambda})m_{0002} + 32D_x m_{0020} + 8D_{2r} - 2C_5 m_{0011} - 2C_2 m_{1001} \quad (16d)$$

$$m'_{1100} = -(\zeta_1 + \zeta\sqrt{\lambda})m_{1100} + C_3P_s m^0_{1000} - C_4 m_{2000} + C_2 m_{1010} + m_{0200} \quad (16e)$$

$$m'_{0110} = -(\zeta_1 + \zeta\sqrt{\lambda})m_{0110} + C_3P_s m^0_{0010} - C_4 m_{1010} + C_2 m_{0020} + m_{0101} \quad (16f)$$

$$m'_{0011} = -(\zeta_2 + \zeta\sqrt{\lambda})m_{0011} - C_5 m_{0020} - C_2 m_{1010} + m_{0002} \quad (16g)$$

$$m'_{1010} = m_{1001} + m_{0110} \quad (16h)$$

$$m'_{0101} = -(\zeta_1 + \zeta_2 + 2\zeta\sqrt{\lambda})m_{0101} + 8D_x m_{1010} + C_3P_s m^0_{0001} - C_5 m_{0110} - C_4 m_{1001} - C_2 m_{1100} + C_2 m_{0011} \quad (16i)$$

$$m'_{1001} = -(\zeta_2 + \zeta\sqrt{\lambda})m_{1001} - C_5 m_{1010} - C_2 m_{2000} + m_{0101} \quad (16j)$$

The steady-state response is determined by setting the left sides to zero and solving for the second moments. It is also possible to determine from these equations the mean-square responses of the uncoupled modes by setting $C_2 = 0$. The steady-state response for uncoupled modes is

$$m_{2000} = 4D_{1r}/\{C_4(\zeta_1 + \zeta\sqrt{\lambda}) - D_x\} \quad (17a)$$

$$m_{0200} = C_4 m_{2000} \quad (17b)$$

$$m_{0020} = 4D_{2r}/\{C_5(\zeta_2 + \zeta\sqrt{\lambda}) - 16D_x\} \quad (17c)$$

$$m_{0002} = C_5 m_{0020} \quad (17d)$$

This solution is stable in the mean square if the following conditions are satisfied

$$D_x/\zeta_1 < \pi^4(1 + R_x^0/\pi^2)[1 + (\zeta\sqrt{\lambda})/\zeta_1]$$

$$4D_x/\zeta_2 < \pi^4(4 + R_x^0/\pi^2)[1 + (\zeta\sqrt{\lambda})/\zeta_2] \quad (18)$$

These conditions depend on aerodynamic pressure, structural

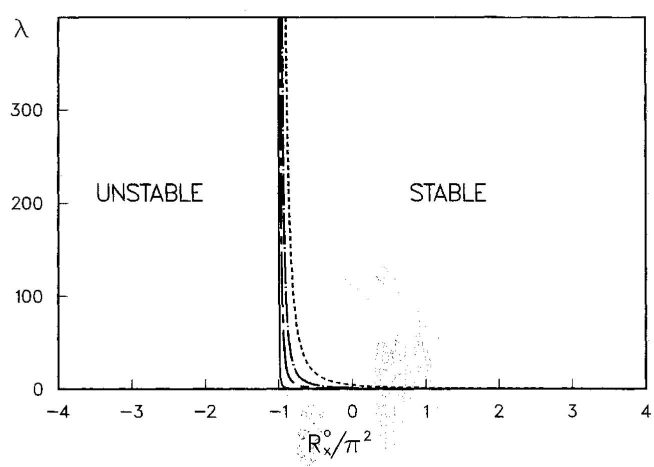

Fig. 2 Mean-square stability boundaries of the first mode for various values of spectral density of in-plane random excitation. —— $D_x = 1.0$, — – — = 5.0, — · — = 10.0, ---- = 20.0, $\zeta_1 = 0.0$, $\mu/M = 0.01$.

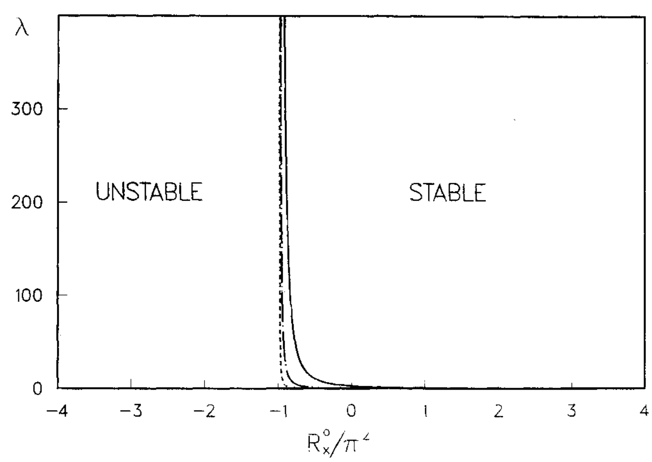

Fig. 3 Mean-square stability boundaries of the first mode for various values of mass ratio parameter. —— $\mu/M = 0.001$, — · — = 0.01, ---- = 0.1, $\zeta_1 = 0.0$, $D_x = 5.0$.

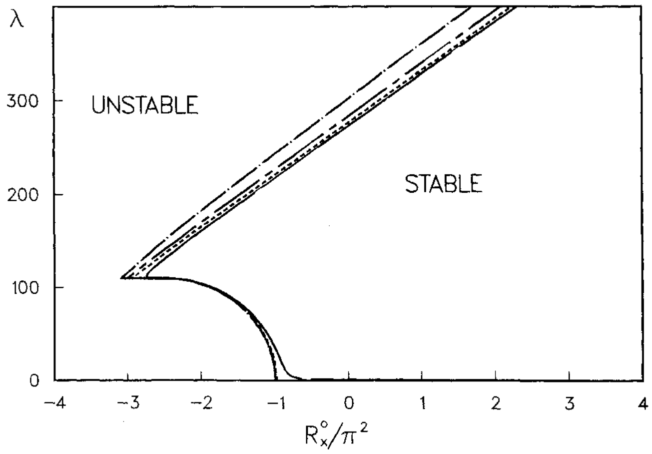

Fig. 4 Mean-square stability boundaries for various structural damping ratios. —— $\zeta_1 = \zeta_2 = 0.0$, ---- = 2, — — — = 5, — · — = 10, $D_x = 5$, $\mu/M = 0.01$.

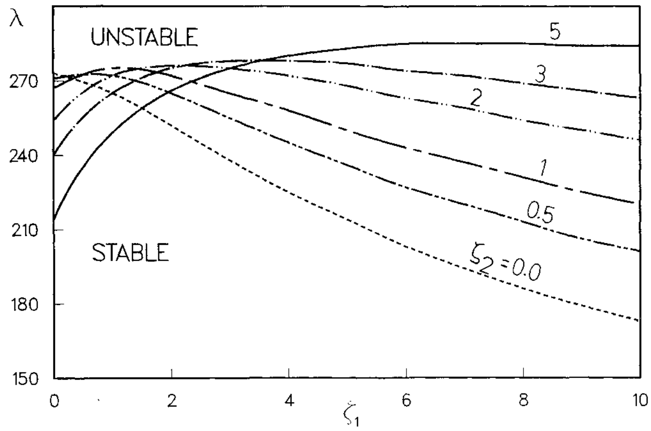

Fig. 6 Mean-square stability boundaries of coupled modes as a function of first mode damping for various damping ratios of the second mode.

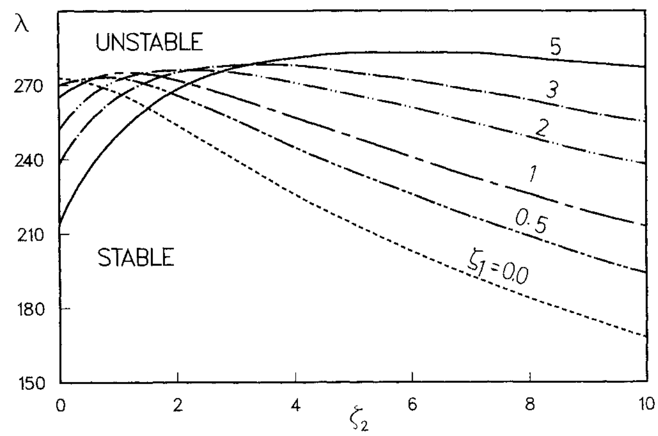

Fig. 5 Mean-square stability boundaries of coupled modes as a function of second-mode damping for various damping ratios of the first mode.

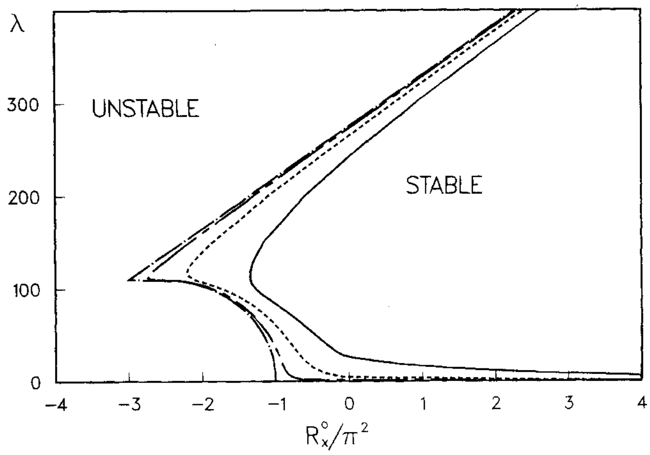

Fig. 7 Mean-square stability boundaries for various values of random in-plane spectral density. — · — $D_x = 0$, — — — = 5, ---- = 20, —— = 50, $\zeta_1 = \zeta_2 = 0$, $\mu/M = 0.01$.

damping coefficients, static in-plane loading R_x^0, spectral density of random in-plane loading D_x, and mass ratio μ/M parameter. The stability boundaries are examined for various values of ζ_1, ζ_2, D_x, and μ/M. Samples of stability boundaries of the first mode are shown in Figs. 2 and 3 for various values of in-plane, random-excitation, spectral-density, and mass-ratio parameter, respectively. It is clear that the mass ratio stabilizes the panel, excitation, and spectral density have a destabilizing effect. The effect of structural damping (not shown) is found to stabilize the panel in the mean square sense.

The stability boundaries of the steady-state response of coupled modes are obtained by considering the coupled 10 differential equations of Eqs. (16). For zero-static and random-pressure components, the stability of the trivial solution of Eqs. (16) is obtained by using the relation of Eq. (13) where $m^0_{k1k2k3k4} = 0$. The results are plotted on the plane of dynamic pressure parameter λ vs static, in-plane load R_x^0/π^2 for various structural damping ratios ζ_1 and ζ_2, parametric, random-load spectral density D_x, and air-to-structure mass ratio μ, (see Figs. 4 through 8). The general trend of these stability boundaries is similar to those predicted by the deterministic theory. For example, Fig. 4 shows that equal, structural, modal-damping coefficients reduce the region of mean-square instability. On the other hand, Figs. 5 and 6 show that unequal modal damping coefficients have a nonbeneficial effect on the stability regions for $D_x = 5$, $\mu/M = 0.01$ and $R_x^0 = 0$. It is seen that if ζ_1 is held constant, the plate is stabilized in the mean-square sense as ζ_2 increases up to a value very close to ζ_1 above which the instability region increases. One also may deduce from the peaks of these curves that there is a critical pair of damping ratios above which the damping becomes nonbeneficial to the plate response. This destabilizing effect of structural damping was also observed in the deterministic theory of flutter.[18-21]

It should be noticed that the structural damping expressed in Eq. (1) by $c\partial^5 w/\partial t \partial x^4$ is identical to the one adopted by Dzygadlo and Kaliski[7] and Dzygadlo[8] and is different from those used by Dugundji[19] and Lottati.[20] Thus the results of Fig. 4 agree with those predicted by Movchan,[21] Bolotin,[22] and Parks[23] only when the two damping ratios are equal. As observed by Flax,[24] the real parts of the roots of the stability equation (for a panel under no random loading) are algebraically additive for small values of combined viscous and viscoelastic damping. For aeroelastic structures such as the aileron-tab system, Done[18] reported that the flutter speed decreases as damping increases. The effect is only small when practical values of purely structural damping were considered. His analysis shows that the fact that the flutter speed falls in some cases when the damping is increased is mainly attributed to a change in the flutter frequency. If the frequency was somehow maintained constant, then addition of damping would always cause the flutter speed to rise. The work of others[25] indicates that the flutter speed always increases when the damping is added to each mode in the same proportion.

Figure 7 shows the effect of in-plane spectral density level on the stability boundaries. As expected in parametric random vibration, the excitation-spectral density reduces the stability region. The effect of the mass ratio parameter μ/M is always stabilizing the panel as shown in Fig. 8.

B. Nonlinear Response

As shown by Ibrahim,[26] damping is not always a favorable source for dynamic stability. The role of nonlinearity is known to bring the unstable system into a bounded, limit cycle especially if the system is parametrically excited. In the present case, the nonlinear response and its stability are determined by including all moment terms that arise from the nonlinear terms (terms with coefficient C_1 in Eq. 10). These terms cause infinite

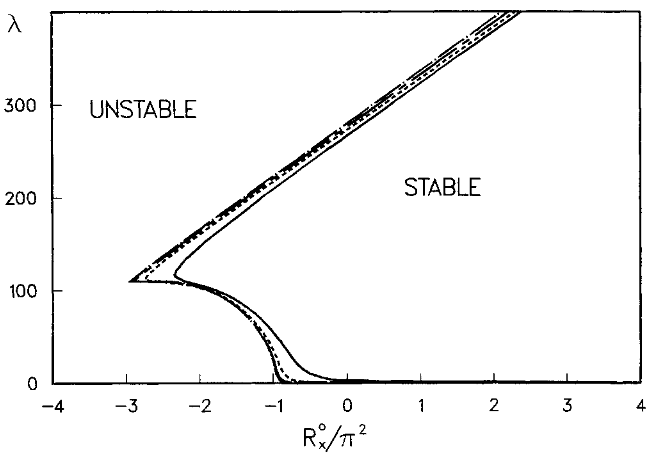

Fig. 8 Mean-square stability boundaries for various values of mass ratio. —— $\mu/M=0.001$, ---- $=0.01$, — — — $=0.05$, — · — $=0.1$, $D_x=5$, $\zeta_1=\zeta_2=0$

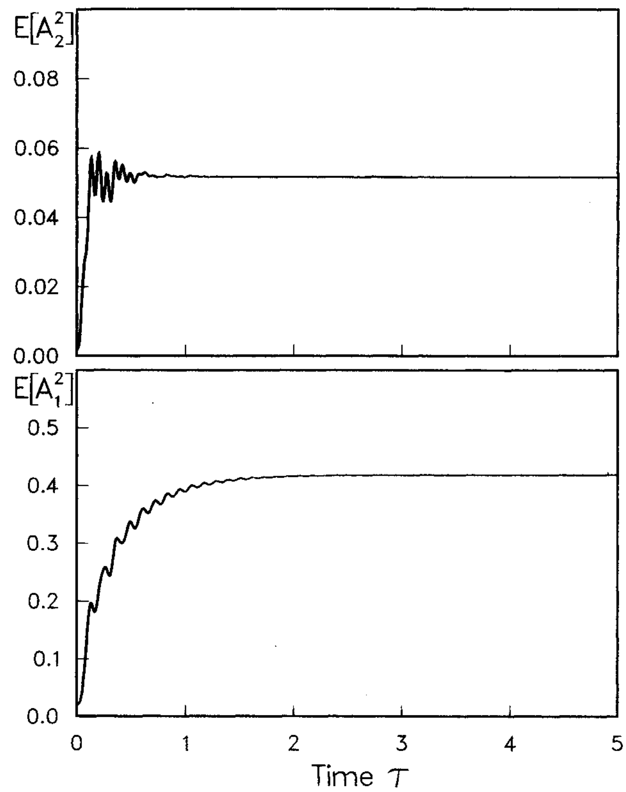

Fig. 10 Time history record of mean-square response for structural damping parameters $\zeta_1=1$ and $\zeta_2=5$. ($D_x=5$, $D_{1r}=D_{2r}=100$, $R_x^0/\pi^2=0.0$, $\mu/M=0.01$, $\lambda=250$)

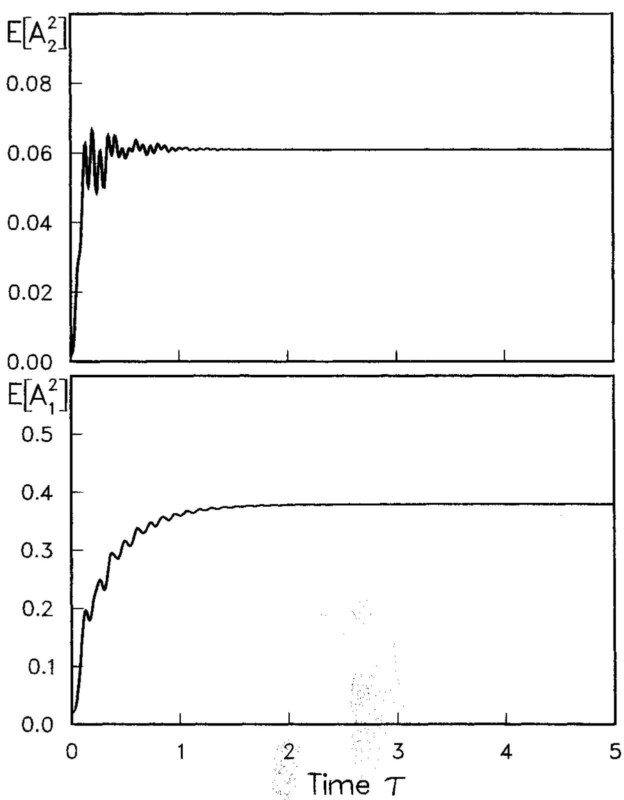

Fig. 9 Time history record of mean-square response for structural damping parameters $\zeta_1=1.0$ and $\zeta_2=3.0$. ($D_x=5$, $D_{1r}=D_{2r}=100$, $R_x^0/\pi^2=0.0$, $\mu/M=0.01$, $\lambda=250$)

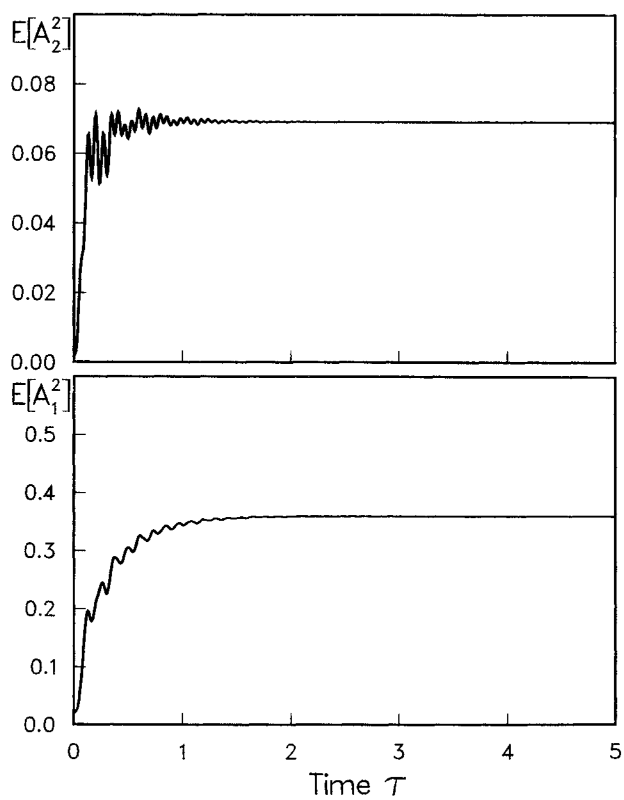

Fig. 11 Time history record of mean-square response for structural damping parameters $\zeta_1=1$ and $\zeta_2=2$. ($D_x=5$, $D_{1r}=D_{2r}=100$, $R_x^0/\pi^2=0.0$, $\mu/M=0.01$, $\lambda=250$)

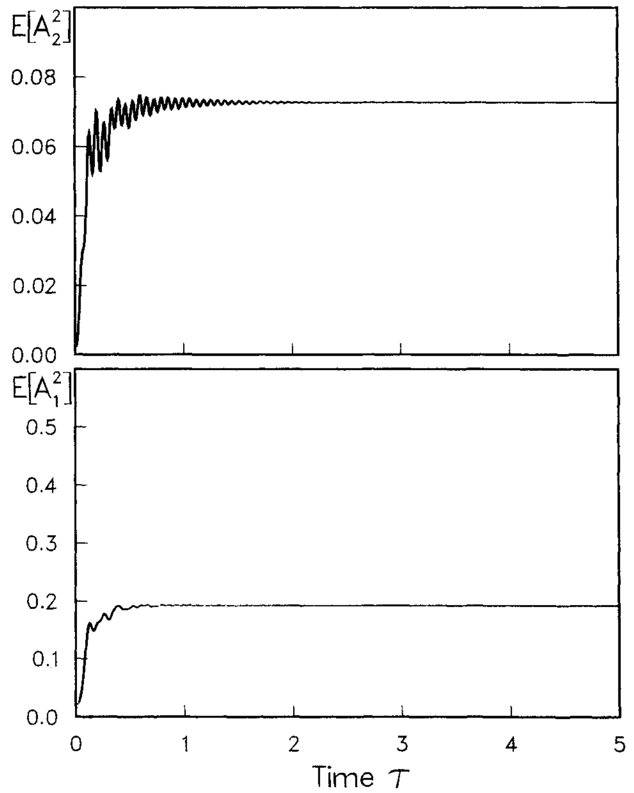

Fig. 12 Time history record of mean-square response for structural damping parameters $\zeta_1 = 4$ and $\zeta_2 = 2$. ($D_x = 5$, $D_{1r} = D_{2r} = 100$, $R_x^0/\pi^2 = 0.0$, $\mu/M = 0.01$, $\lambda = 250$)

coupling between moment equations, i.e., moment equations of order N are coupled with moments of order $N+2$. As a first step of approximation, the second-order moment equations will be closed via a Gaussian closure scheme based on the properties of cumulants. A cumulant is a statistical parameter,[17] and if it is of order greater than two, its value gives a measure to the deviation of the response process from normality. In this analysis it will be assumed that the response will not depart significantly from normal distribution, and all cumulants of order greater than 2 are set to zero. Since all cumulants can be expressed in terms of moments of corresponding and less orders,[17] one can express moments order $N+2$ in terms of moments of order $N+1$ and less. For example, if third and fourth cumulants are set to zero, the corresponding third and fourth moments are given by the following expressions.[17]

$$E[X_iX_jX_k] = \sum^{3} E[X_i]E[X_jX_k] - 2E[X_i]E[X_j]E[X_k] \tag{19a}$$

$$\begin{aligned} E[X_iX_jX_kX_\ell] &= \sum^{4} E[X_i]E[X_jX_kX_\ell] \\ &- 2\sum^{6} E[X_i]E[X_j]E[X_kX_\ell] + \sum^{3} E[X_iX_j]E[X_kX_\ell] \\ &+ 6E[X_i]E[X_j]E[X_k]E[X_\ell] \end{aligned} \tag{19b}$$

where numbers over summation signs refer to the number of possible terms generated in the form of the indicated expressions without allowing permutation of indices.

The first- and second-moment equations are closed via this approach and solved by numerical integration by using the International Mathematical and Statistical Library (IMSL) subroutine DIVPRK (Initial Value Problem Solver, Runge-Kutta-Verner Fifth and Sixth Order Numerical Integration Method) on the SUN 3/260C computer. Contrary to the nonstationary response of dynamic systems with nonlinear inertia

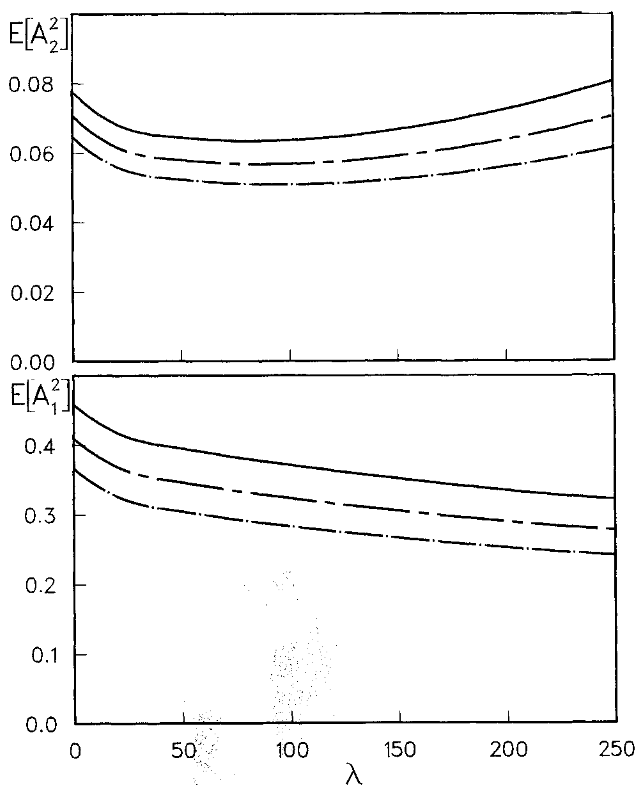

Fig. 13 Dependence of mean-square response on aerodynamic pressure parameter λ for three different in-plane loads. —— $R_x^0/\pi^2 = -1$, — — — = 0.0, —·— = 1.

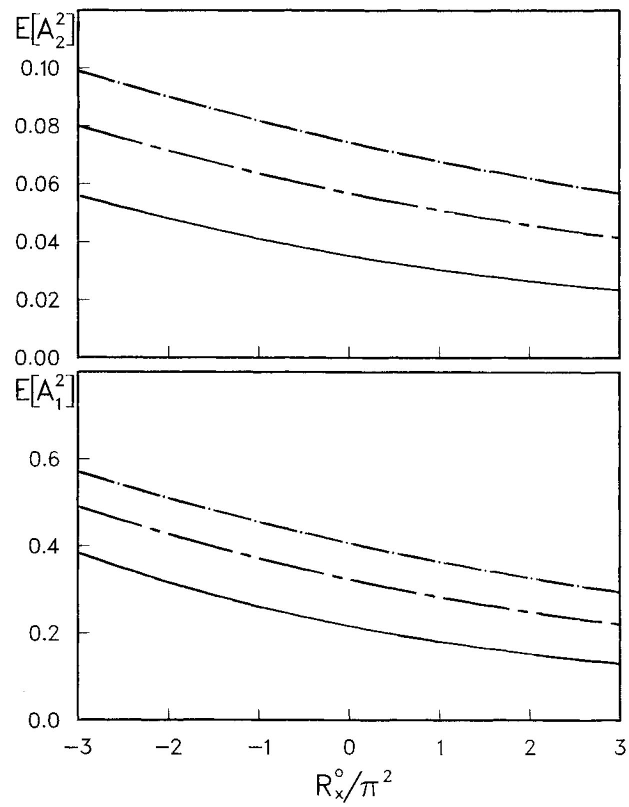

Fig. 14 Dependence of mean-square response on in-plane, static loading for three different random pressure densities. —— $D_{1r} = D_{2r} = 50$, — — — = 100, —·— = 150.

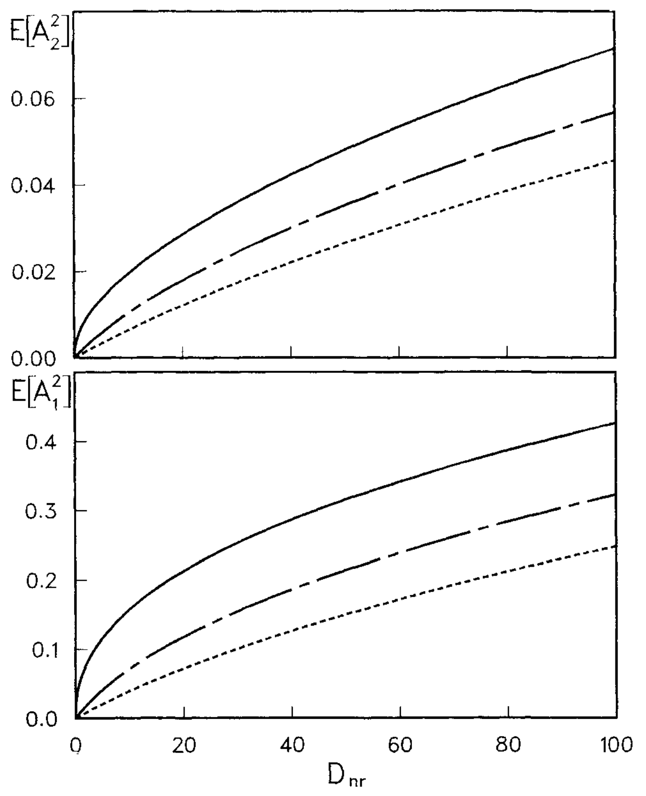

Fig. 15 Dependence of mean-square response on random pressure densities for different in-plane loads. —— $R_x^0/\pi^2 = -2$, — — — = 0.0, ----- = 2.

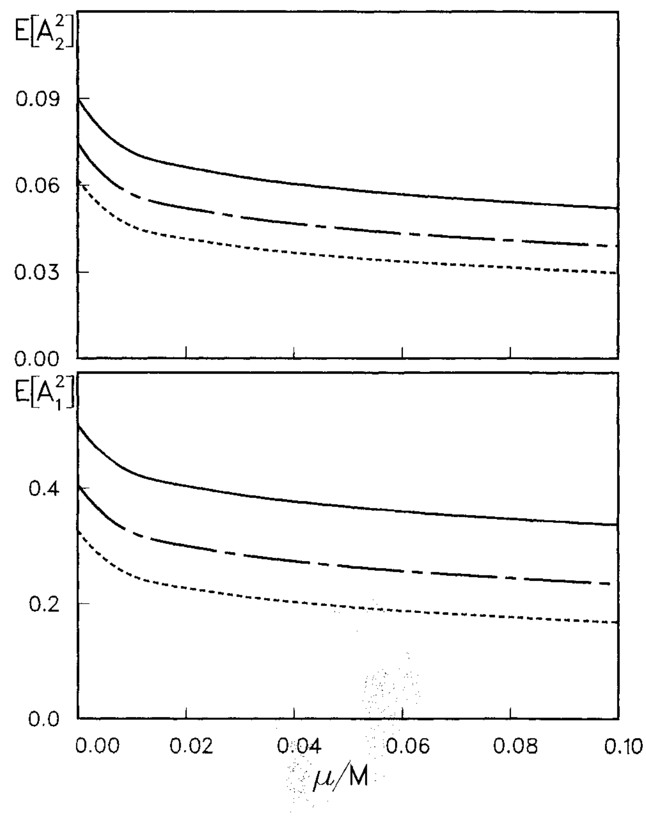

Fig. 16 Dependence of mean-square response on mass ratio parameter for different in-plane loads. —— $R_x^0/\pi^2 = -2$, — — — = 0.0, ----- = 2.

coupling,[27,28] the numerical integration shows that all response statistics reach a stationary state after a finite transient period over which the response statistics fluctuate. The steady-state value of the response statistics are the same for different initial conditions. The effect of initial conditions appears only in the transient period. Figures 9 and 10 show time history response records for unequal damping modal ratios $\zeta_1, \zeta_2 = (1,3)$ and (1,5), respectively. An important feature is that any increase of second mode damping will remarkably reduce the mean-square response of the second mode and will result in a slight increase in the mean square of the first mode. Figures 11 and 12 are obtained for $\zeta_1, \zeta_2 = (1,2)$ and (4,2) respectively. The effect is reversed from those shown in the previous two records.

The effect of aerodynamic pressure on the steady-state, mean-square response is shown in Fig. 13 for different values of in-plane static force $R_x^0/\pi^2 = -1$, 0.0, 1. It is obvious that the aerodynamic pressure has opposite effects on the response of the two modes. Furthermore, the tensile, in-plane load always reduces the response mean squares of the two modes as indicated in Fig. 14. The mean-square response of the first mode decreases as the aerodynamic pressure increases and the second mode increases. The effect of the spectral-density level of the random component of the normal pressure loads on the response is the same for both modes as shown in Fig. 15. The dependence of the mean-square response upon the spectral density is almost nonlinear only if the static, in-plane load is compressive; otherwise the relationship is linear. Figure 16 shows that the mass-ratio parameter reduces the mean-square response of the two modes.

V. Discussion and Conclusions

The stochastic flutter of two-dimensional panels in supersonic flow is examined by using the Markov field approximation theory. The in-plane loading and aerodynamic pressure include random components. The analysis includes the two-mode interaction and a Gaussian-closure scheme. Accurate results are obtained if one includes many modes and a non-Gaussian representation of the nonlinear response. In the case of deterministic theory of panel flutter, it is possible to solve up to six mode interactions by numerically solving the corresponding six nonlinear differential equations. However, in the random case, several difficulties arise if one considers more than two modes. For example, if n is the number of state coordinates (twice the number of modes), the number K of differential equations of response moments of order N is given by the relationship[17]

$$K = n(n+1)(n+2)\dots(n+N-1)/N!$$

It is obvious that the number of equations increases drastically if a non-Gaussian closure is considered. Table 1 gives the number of moment differential equations needed for Gaussian (first- and second-order moments) and non-Gaussian (first-through fourth-order moments) closures.

One may recall that the deterministic theory of panel flutter first started with the study of two-mode interaction. This theory has witnessed a remarkable progress which included more modes and new phenomena.[1,2] However, few attempts[6,13] based on Monte Carlo simulation have been made in the ran-

Table 1 Number of moment equations for Gaussian and non-Gaussian closures corresponding to different number of modes

No. of Modes	Gaussian	Non-Gaussian
2	14	69
3	27	209
4	44	494
5	65	1000

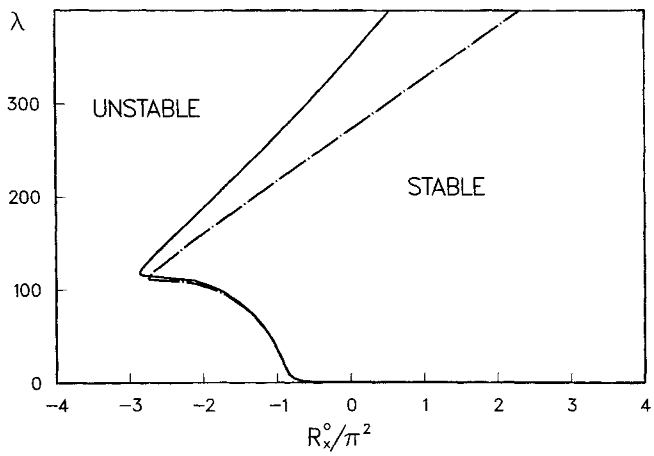

Fig. 17 Comparison of mean-square stability boundaries for two- and three-mode interactions. — · — 2 modes, —— 3 modes, $\zeta_1 = 0.0$, $D_x = 5$, $\mu/M = 0.01$.

dom theory of panel flutter. The results reported in this paper are based on Markov theory and constitute the first part of a series of papers. Parts II and III will consider up to three modes by using Gaussian and non-Gaussian closures.

A sample of the early results is shown in Fig. 17, which includes two stability boundaries based on two modes and three modes. It is clear that the stability boundary of two-mode interaction is more conservative than the one obtained for three modes. Professor Librescue (private communication) of Virginia Polytechnic Institute indicated to the first author that two-mode flutter is more conservative provided the two modes are consecutive.

The assumption of near-normality used in the nonlinear response analysis provides excellent approximation in determining the response statistics in regions well remote from the stability boundaries. The Gaussian closure provides reasonable results when compared with those predicted by non-Gaussian closures.[27]

The stability and response of first moments correspond to the case of static buckling. For the case of second moments, the stability boundaries are obtained in terms of aerodynamic pressure parameter λ, static, in-plane load component R_x^0/π^2, parametric, random-load spectral density D_x, air to structure mass ratio μ, and modal damping coefficients ζ_1 and ζ_2. When the structural modes have unequal damping, the increase of damping of any mode is found to have a destabilizing effect. The effect is only stabilizing when the two modes have equal damping ratios. The steady-state mean-square response is also affected by the damping ratios. The effect is that if the damping ratio of the first mode is kept constant, then any increase of the second-modal damping results in a slight increase in the response of the first mode and a corresponding decrease of the second-mode response statistics.

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